1999 2U TRANS TRIAL.

Q1.

(a) Solve $\frac{2x-1}{3} - \frac{5x-3}{6} = 2$

(b) Evaluate to 2 decimal places $e^x \sin 2x$ when $x = 2$.

(c) Write $7.\dot{2}\dot{1}$ as a mixed fraction.

(d) A person deposits $650 into a savings account for 10 years at an interest rate of 4.35 % p.a.

Find the value of the account at the end of 10 years.

(e) Rationalise the denominator and simplify $\frac{7-\sqrt{2}}{3+\sqrt{2}}$.

Question 2 (Start A New Page)

(a) Differentiate y wrt x :

(i) $y = e^2 \pi$

(ii) $y = x \sec x$

(iii) $y = 2 \ln (x^3 + 4)$

(iv) $y = \tan^3 2x$

(b) Find the equation of the tangent to the curve $y = \frac{x}{x^2+1}$ at $x = 2$.

Write your answer in general form.

Question 3 (Start A New Page)

(a) Find the primitive function of $\frac{e^{3x}}{3} - \cos 2x$.

(b) Find :

(i) $\int (7x-3)^2 \, dx$

(ii) $\int \sqrt{x}\,(2x+1)\, dx$

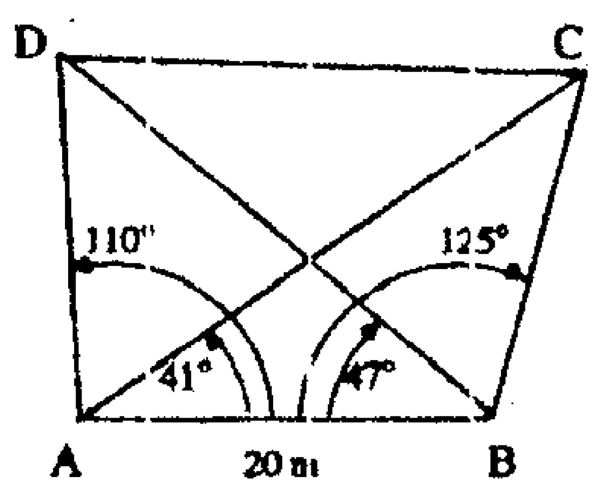
(iii) $\int \frac{e^{2x}+1}{e^x}\, dx$

(c) Evaluate (i) $\int_0^{\frac{\pi}{4}} 4 \sin 2x \, dx$

(ii) $\int_0^3 \frac{2x}{4x^2+1}\, dx$

Question 4 (Start A New Page).

(a) A surveyor records the following diagram.



$\angle ABC = 125°$, $\angle ABD = 47°$, $\angle BAD = 110°$, $\angle BAC = 41°$

Find the lengths (2 decimal places) of :

(i) BD

(ii) CD if BC = 54 metres.

(b) Given A(−5 , 4) , B(3 , 7) , C(9 , 1) and D(−1, −4)

(i) Show (α) ABCD is a trapezium

(β) AD⊥CD

(ii) Find the area of ABCD

Question 5 (Start A New Page)

(a) Graph $y = \frac{x+3}{x-2}$

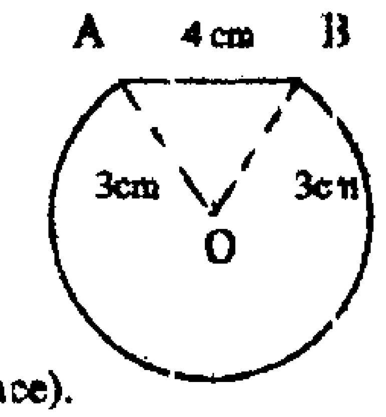
(b) Simplify $1 + \frac{5}{x-2}$

(c) The region bounded by the curve $y = \frac{x+3}{x-2}$ and the lines $x = 3$, $x = 4$, $y = 0$, is rotated around the x axis . Find the volume of revolution.

(d) Find the area bounded by the curve $y = \sqrt{9-x^2}$ and the positive x and y axes .

Question 6 (Start A New Page)

(a) The cross sectional area of a circular cam centre O is shown.



Show that $\angle AOB = 1.459$ radians,

hence find (i) the perimeter of the cam (to 1 decimal place)

(ii) the cross sectional area of the cam (to 1 decimal place).

(b) A compound substance decomposes into it's elements at a rate proportional to the amount of compound present , ie $\frac{dM}{dt} = -kM$ where M is the mass of the compound present after t seconds and k is the proportionality constant.

(i) Show $M = A e^{-kt}$ is a solution of the differential equation where A is a constant.

(ii) If 900 grams decomposes to 850 grams in 50 seconds

(α) show A = 900 and $k = \frac{1}{50} \ln\left(\frac{18}{17}\right)$,

(β) find the amount compound (nearest gram) remaining after 140 seconds .

Question 7 (Start A New Page).

Given the function $y = \dfrac{10}{3 + 2\sin x}$ in the domain $0 \le x \le 2\pi$

(a) find the location of all the stationary points and determine their nature

(b) graph the function in the given domain

(c) using 5 function values evaluate by Simpson's rule $\displaystyle\int_0^{2\pi} \frac{10\,dx}{3 + 2\sin x}$.

Question 8 (Start A New Page).

(a) A man borrows \$ 13500 at 9.9% p.a. reducible monthly interest for 4 years, and agrees to repay the loan in equal monthly instalments. Find the value of the monthly repayment .

(b) Twelve students are to be chosen from twenty students of equal ability.

Find the probability that :

(i) three particular students A , B , C are chosen

(ii) students A and B are chosen but student C is not chosen

(iii) none out of students A , B , or C are chosen

(iv) at least one of the students A , B , or C are chosen.

Question 9 (Start A New Page)

(a) Find the values of k to make $y = (k + 2)x^2 + 4\sqrt{3}x + 5 - k$ positive definite.

(b) Given the function $g(x) = \dfrac{e^x - e^{-x}}{2}$,

(i) show that g(x) is an increasing function for all values of x

(ii) find the minimum value of the gradient of g(x) and where this occurs.

(iii) graph $y = g(x)$

Question 10 (Start A New Page).

(a) A particle moves with velocity v m/s in time t seconds according to :

$$v = \frac{6}{\sqrt{t+1}}$$

Find (i) the acceleration as a function of time t

(ii) the displacement x as a function of time t if initially the particle was 2 metres to the right of the origin.

(b) In a pentagon ABCDE , AB = AE , BC = ED and BD = EC.

Prove that AC = AD

JRAHS 2U 1999 TRIAL

QUESTION 1

(a) $x = -11$

(b) -5.59

(c) $7\frac{7}{33}$

(d) \$995.03

(e) $\frac{23-10\sqrt{2}}{7}$

QUESTION 2

(a)(i) 0

(ii) $x\sec x\tan x + \sec x$

(iii) $\frac{6x^2}{x^3+4}$

(iv) $6\tan^2 2x . \sec^2 2x$

(b) $3x + 25y - 16 = 0$

QUESTION 3

(a) $\frac{1}{9}e^{3x} - \frac{5}{2}\sin 2x + c$

(b) (i) $\frac{1}{21}(7x-3)^3 + c$

(ii) $4/5\, x^{5/4} + 2/3\, x^{3/2} + c$

(iii) $e^x - e^{-x} + c$

(c) (i) 2

(ii) $\frac{1}{4}\ln 37$

QUESTION 4.

(a)(i) 48.10 m

(ii) 64.42 m

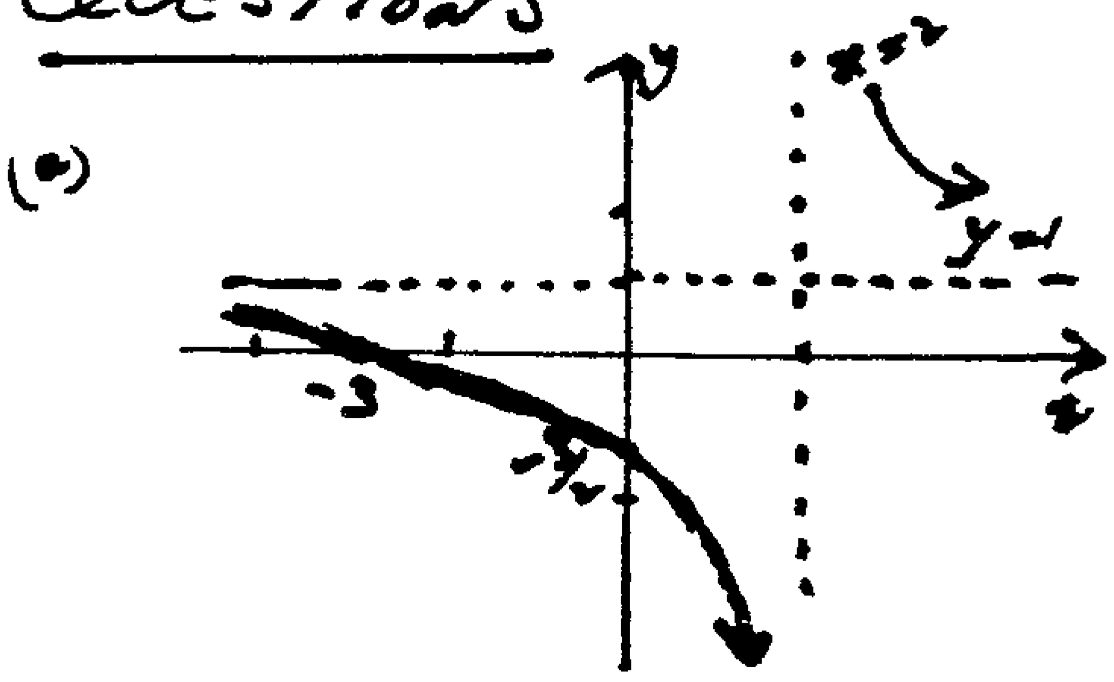
(b) (i)(α) $AB \parallel CD$ ($m = \frac{1}{2}$)

(β) $m(AD) \cdot m(CD) = -1$

(ii) $80\ u^2$

QUESTION 5

(a)



(b) $\frac{x+3}{x-2}$

(c) $\frac{\pi}{2}(27 + 20\ln 2)$

(d) $\frac{9\pi}{4}\ u^2$

QUESTION 6

(a) (i) 18.5 cm

(ii) 26.2 cm^2

(b)(α)(i) —

(ii) —

(β) 767 g.

QUESTION 7

(a) min tp $(\pi/2, 2)$

max tp $(3\pi/2, 10)$

(b) —

(c) $\frac{92\pi}{9}$

QUESTION 8

(a) \$341.75

(b) (i) 11/57

(ii) 44/285

(iii) 14/285

(iv) 271/285

QUESTION 9

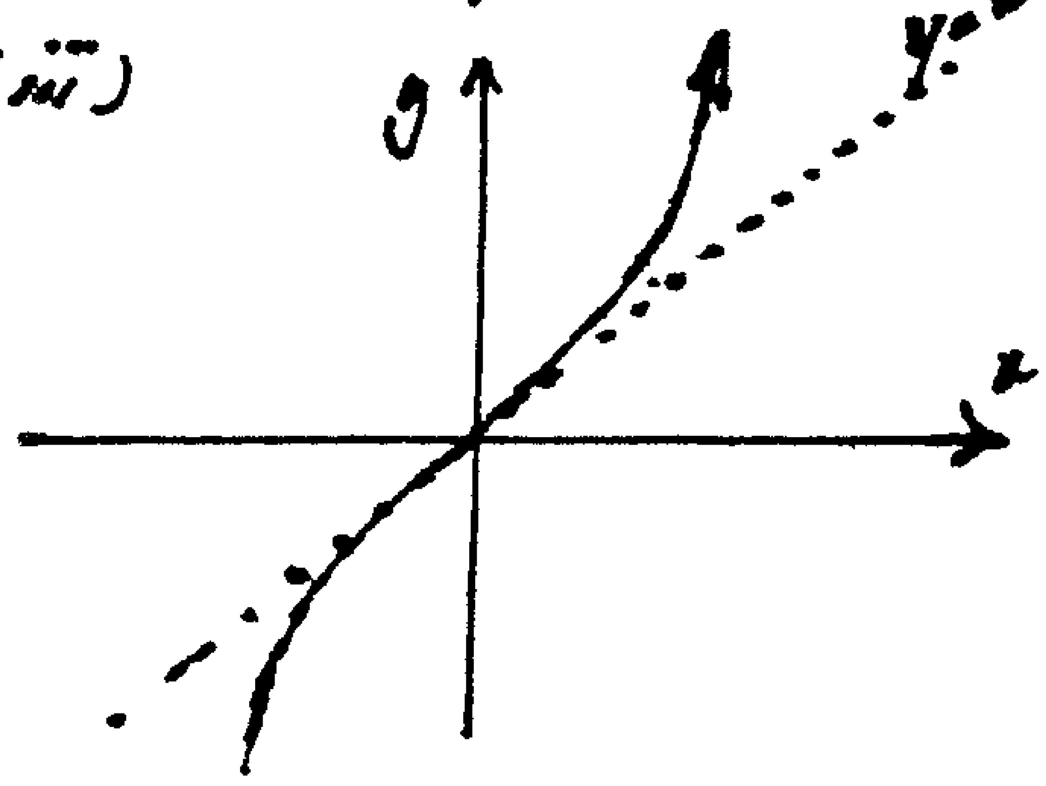
(a) $1 < k < 2$

(b) (i) $g'(x) = \frac{1}{2}(e^x + e^{-x})$

≥ 0 for all x

(ii) min. slope = 1.

(iii)



QUESTION 10

(a)(i) $a = \frac{-3}{(t+1)^{3/2}}$

(ii) $x = 12\sqrt{t+1}$ -1

(b) —